

# Distinguished features of muon colliders physics potential

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**Abstract.** Features of muon collider physics potential are briefly discussed.

## INTRODUCTION

The muon collider concept was first mooted many years ago [1–4]. The main impetus for such an idea was the craving to get rid of the powerful synchrotron radiation from which cycling electron-positron colliders suffer so much. Muons being as much as  $\approx 200$  times heavier than electrons undergo this disease to a much less extent because the intensity of the synchrotron radiation is proportional to  $mass^{-4}$ . The interest of last several years to the muon colliders has hugely grown due to several reasons. From the point of view of the machine design consideration it was revealed that the ionization cooling concept [3,4] offers the possibility of making a high luminosity accelerator. On the other hand, the physics potential of the muon collider was enriched by the possibility to build a "Higgs boson factory" in analogy with existing " $Z^0$  boson factories", and, therefore, such a facility might provide a unique laboratory for particle physics research (for a recent review of both the machine design and physics potential see e.g. Ref. [5]).

Apparently one of the main objectives of the present-day elementary particle physics is to find the Higgs boson and investigate its properties with as high precision as possible. It is hoped that this particle will be revealed at the forthcoming LHC machine. However, there exists the so-called intermediate mass region, extended from  $m_H > 95$  GeV to  $m_H \leq 2m_Z$ , which is the most difficult for experimental research. In spite of this difficulty it is hoped that LHC will allow to know the Higgs mass with the precision sufficient to tune to the resonance with the muon collider. Happily, just at this region the SM Higgs boson has a narrow width, more exactly, only if its mass is less then the threshold of  $W^\pm$  - pair production (see below). Thereby, the  $\mu^+\mu^-$  collider would not merely fill the gap but improve the precision of the studies in this region to a great extent.

## MOTIVATION

As mentioned at the end of Introduction, the Higgs boson width in the SM is narrow only in the region of c.m. energy up to  $\sqrt{s} \sim 2m_W$ , so the Higgs factory is feasible only in this region. To see this let us calculate the cross section of the process  $\mu^+\mu^- \rightarrow W^+W^-$  which proceeds only through the Higgs boson exchange in the s-channel. Using the standard notations, the result for this cross section reads as follows:

$$\sigma_{\mu^+\mu^- \xrightarrow{H} W^+W^-} = \frac{\pi\alpha^2}{\sin^4\theta_W} \frac{m_\mu^2}{16} \frac{s - 4m_\mu^2}{(m_H^2 - s)^2 + \Gamma_H^2 m_H^2} \left( \frac{1}{2} \frac{s^2}{m_W^4} - 2 \frac{s}{m_W^2} + 6 \right). \quad (1)$$

It can be seen from the equation above that the cross section due to the Higgs boson exchange reaches its maximum,  $\sigma_{max} \cong 0.067 pb$  at the c.m. energy  $\sqrt{s} \sim 2m_Z$  while the "conventional" cross section (due to  $\gamma$ ,  $\nu$  and  $Z^0$  exchange) reaches at this point the value of  $\approx 15$  pb. Partly in view of this it is expedient to search for other processes where the interaction of the Higgs within the lepton sector would be involved. Two examples of that consideration was presented in the paper [6], where processes  $\mu^+\mu^- \rightarrow HZ^0$  and  $\mu^+\mu^- \rightarrow H\gamma$  were proposed as complementary to that of the resonance Higgs scalar production. Note that the latter of the two processes above is negligibly small at the tree level in the case of electron-positron collision, but there is hope to observe it at the muon collider.

## ASSOCIATED $HZ$ PRODUCTION IN SM

Let us begin with the Bjorken process having muons as the initial state particles,  $\mu^+\mu^- \rightarrow ZH^0$ .

Usually in the course of cross section calculations one uses the s-channel diagram alone. Let us do it and take into account masses of initial muons. Then we obtain the following asymptotics of this process at  $\sqrt{s} \rightarrow \infty$ :

$$\sigma_{\mu^+\mu^- \rightarrow ZH^0}^{(s-channel),as} |_{m_\mu \neq 0} = \frac{2\pi\alpha^2}{\sin^4(2\theta_W)} \cdot g_A^2 \cdot \frac{m_\mu^2}{m_Z^4}. \quad (2)$$

It can be seen that despite the fact that this diagram is pure s-channel one, the corresponding cross section is not falling with energy but approaches a constant limit, whose value is equal to  $\cong 1.2 \cdot 10^{-2} fb$ . Concerning the angular dependence of this cross section, it could be seen that this distribution is **flat**, indicating that it comes entirely from the  $J = 0$  partial wave. It is obvious that this behaviour contradicts the unitarity condition which requires  $\sigma_{J=0} \leq s^{-1}$  at high energy.

Now we calculate the contribution to this cross section given by the cross channels diagrams, t- and u- ones. It turns out that the corresponding contribution is again equal exactly to the value of  $\cong 1.2 \cdot 10^{-2} fb$ . The corresponding

angular distribution is also flat. At last, let us take into account the interference term between the two classes of the diagrams above. We found that it is equal exactly to  $\cong -2.4 \cdot 10^{-2} fb$ . Adding all three contributions we obtain the result which removes a seeming contradiction. As it must be, the asymptotic form of the cross section for the process under consideration at  $\sqrt{s} \rightarrow \infty$  acquires the "desired" form, i.e. it falls with energy

$$\sigma_{\mu^+\mu^- \rightarrow H^0 Z}^{as} = \frac{1}{3} \cdot \frac{\pi \alpha^2}{\sin^4(2\theta_W)} \cdot (g_V^2 + g_A^2) \cdot \frac{1}{s}. \quad (3)$$

Attention should be drawn to the difference between factors containing the coupling constants entering into Eqs. (2) and (3). The obtained cancellation reflects the most fundamental property of the electroweak theory. This is a consequence of the unitarity condition at the tree level which must be fulfilled in any non-Abelian gauge theory with the symmetry broken in a manner like the Higgs mechanism [7].

In order to extract information about the Higgs – lepton sector interplay let us look once more at the individual contributions to the discussed cross section. In this respect it is worthwhile to note that all three contributions reach their constant asymptotic values not simultaneously. Those stemming from the sum of the t-channel and u-channel go to the plateau at the energy around 1 TeV. The negative contribution reaches its minimum value at  $\sqrt{s} \cong 2.5 TeV$ , while the cross section, corresponding to the s-channel, becomes constant (at finite muon mass) far away from 1-2 TeV region. However, in spite of different characters of the behaviour of these contributions, it seems unlikely that this difference will be observed experimentally owing to the small value of the  $\mu$ -meson mass. Partly because of that, in the next paragraph we will consider the process where the Higgs-Gauge-Boson vertex is not involved.

## ASSOCIATED $H^0\gamma$ PRODUCTION. *ARBITRARY ENERGIES CASE*

We now turn to a process analogous to the one just considered but which is free from the s-channel diagram complication. At first, it is instructive to consider the arbitrary energies case which covers the low energy region where scalar and pseudoscalar couplings of the Higgs with muons yield different cross sections as opposed to the high energy case. In addition, the expressions for cross sections obtained are very simple. We omit here differential cross sections and give only the integrated expressions. For the scalar case, we have obtained the following cross section without neglecting masses of the initial particles

$$\sigma_{\mu^+\mu^- \rightarrow H^0\gamma}^{Scalar} = \frac{\pi \alpha^2}{2 \sin^2 \theta_W} \frac{m_\mu^2}{M_W^2} \frac{1}{s^2} \frac{1}{\beta} \frac{1}{s} \frac{1}{s - m_H^2} \times$$

$$\left\{ -2m_H^2 s \beta_H^2 + (s^2 \beta^4 + m_H^4 \beta_H^2) \frac{1}{\beta} \ln \frac{1+\beta}{1-\beta} \right\}, \quad (4)$$

where in addition to the usual  $\beta = \sqrt{1 - \frac{4m_\mu^2}{s}}$  we have introduced the notation  $\beta_H = \sqrt{1 - \frac{4m_\mu^2}{m_H^2}}$  with  $\sqrt{s}$  as the c.m. energy. For the case of the pseudoscalar coupling of the Higgs with the muon, we have

$$\sigma_{\mu^+\mu^- \rightarrow H^0\gamma}^{Pseudoscalar} = \frac{\pi\alpha^2}{2\sin^2\theta_W} \frac{m_\mu^2}{M_W^2} \frac{1}{s^2} \frac{1}{\beta} \frac{1}{s - m_H^2} \times \left\{ -2m_H^2 s + (s^2 + m_H^4 \beta_H^2) \frac{1}{\beta} \ln \frac{1+\beta}{1-\beta} \right\}. \quad (5)$$

It is the  $\gamma_5$  non-invariance, which is responsible for the different behaviours of the two cross sections, see eqs. (4) and (5), at low energies. The formulae obtained are very important, for example, in searching for the light axion or even light Higgs at  $\Upsilon$  decays to the  $\gamma$  + nothing, i.e. in the  $b\bar{b}$  annihilation, in which case the cross section is greatly enhanced due to the large mass of the  $b$ -quark. At higher energies this difference disappears as will be seen in the next section.

## ASSOCIATED $H^0\gamma$ PRODUCTION. HIGH ENERGY CASE.

The differential cross section of the process  $\mu^+\mu^- \rightarrow H\gamma$  for the case, when the photon is hitting a non-forward detector, and when we neglect the muon mass (apart from where it affects the muon-Higgs boson coupling constant) reads as follows:

$$\frac{d\sigma^{\gamma H}}{d(\cos\theta)} = \frac{\pi\alpha^2}{2\sin^2\theta_W} \frac{m_\mu^2}{M_W^2} \frac{s^2 + M_H^4}{s^2(s - M_H^2)} \frac{1}{(1 - \cos^2\theta)}, \quad \cos\theta \leq 1. \quad (6)$$

The corresponding integrated cross section is as follows (we have neglected here the muon mass apart from where it is a part of the muon-Higgs boson coupling constant):

$$\sigma_{\gamma H}^0 = \frac{\pi\alpha^2}{2\sin^2\theta_W} \frac{m_\mu^2}{M_W^2} \frac{1}{s^2} \frac{1}{s - m_H^2} \left[ -2m_H^2 s + (s^2 + m_H^4) \ln \frac{s}{m_\mu^2} \right], \quad (7)$$

where superscript 0 means that the cross section is calculated at the tree-diagram level.

With the yearly integrated luminosity of  $\mathcal{L} \cong 10^3 \text{ fb}^{-1}$  expected at future  $\mu^+\mu^-$  colliders, one could collect 20 to 30  $H^0\gamma$  events (the detector efficiency is supposed equal 1, and the acceptance –  $4\pi$ ). The signal which mainly consists

of a photon and  $b\bar{b}$  pairs in the low Higgs mass range or  $WW/ZZ$  pairs for Higgs masses larger than  $\cong 200$  GeV, is extremely clean. The background should be very small since the photon must be very energetic and the  $b\bar{b}$  or  $WW/ZZ$  pairs should peak at an invariant mass  $M_H$ . Therefore, despite the low rates, a clean signal gives a good possibility to detect these events.

Another mechanism for associated  $\gamma H$  production is the one of the electroweak loops, considered in Ref. [8] and in recently published [9,10]. This last mechanism is equally applicable to the case of colliding  $\mu^+\mu^-$  beams and to the  $e^+e^-$  case. However, the both mechanisms give the cross sections with very different c.m. energy behaviours. As is seen from Eq. 2 the tree-level cross section grows when  $\sqrt{s} \rightarrow m_H$  due to kinematical factor  $\frac{1}{s-m_H^2}$  in front of it. Contrary to this case, the one-loop cross section is negligible at the threshold and rises with energy. Comparative pictures of the two types cross section behaviours are depicted on Fig. 3 of Ref. [10] at some representative Higgs boson mass values. A remarkable feature of these figures is equality of the tree-level and one-loop cross sections at the practically invariable point  $m_H \approx \frac{\sqrt{s}}{2}$ , after which the tree-level cross section falls rapidly and the process is dominated by the one-loop amplitudes, while up to this point the main contribution to the cross section comes from the tree-level graphs. At the first sight this feature delivers a good possibility of studying the Higgs behaviour in lepton sector. But we must realize that the tree-level cross section has the "bad behaviour" in the vicinity of the point where  $\sqrt{s} \gtrsim m_H$ , so we need to take care of this region in order to smooth the front edge of the cross section curve. To this end we need to calculate the radiative corrections (RC) to the process under consideration.

In other words, to make a consistent comparison between the lowest order cross section for the process  $\mu^+\mu^- \rightarrow H\gamma$  and that due to the one-loop amplitudes, we need to calculate quantum electrodynamics (QED) correction to the tree-level amplitudes. That is the main aim of the next section.

## RADIATIVE CORRECTIONS TO THE PROCESS

$$\mu^+\mu^- \rightarrow H\gamma.$$

Due of the lack of the space here only a very concise sketch of RC will be given. For the comprehensive information I refer to [12]. For the process under study 1) the first order leading logarithmic correction were obtained, and 2) the contribution of the higher order perturbation theory were taken into account in the leading logarithmic approximation. It is expedient to give here the expression for the case 2). The master formula for the radiatively corrected cross section has the form of the Drell-Yan cross section. So, we suggest to write the result as a convolution of the modified lepton structure functions with the shifted cross-section of the hard subprocess. It reads

$$\frac{d\sigma}{d\cos\theta} = \int_{z_1^{min}}^1 dz_1 \widetilde{D}(z_1) \int_{z_2^{min}}^1 dz_2 \widetilde{D}(z_2) \frac{d\tilde{\sigma}_0(z_1 p_1, z_2 p_2)}{d\cos\theta} \left(1 - \frac{\alpha}{2\pi} K\right) \Theta(\omega - \omega_{th}) , \quad (8)$$

where a part of non-leading terms is taken into account by the so-called  $K$ -factor (again, see the ref. [12]);  $\omega_{th}$  is the experimental energy threshold of the photon registration. A *smoothed* representation for the modified  $D$ -function was used:

$$\widetilde{D}(z, L) = \frac{1}{2} \beta (1-z)^{\beta/2-1} (1+z^2) (1 + \mathcal{O}(\beta^2)), \quad \beta = \frac{2\alpha}{\pi} (L-1) . \quad (9)$$

The energy conservation law gives us the energy of the detected photon

$$\omega = \frac{sz_1 z_2 - M_H^2}{2\varepsilon(z_1 + z_2 - c(z_1 - z_2))} . \quad (10)$$

The lower limits of integration over  $z_{1,2}$  are to be defined also just from the above expression by imposing the condition  $\omega > \omega_{th}$ :

$$z_1^{min} = \frac{M_H^2 + \sqrt{s}\omega_{th}(1+c)}{s - \sqrt{s}\omega_{th}(1-c)} , \quad z_2^{min} = \frac{M_H^2 + \sqrt{s}z_1\omega_{th}(1-c)}{sz_1 - \sqrt{s}\omega_{th}(1+c)} . \quad (11)$$

In Fig. 1 we presented the values of radiative corrections as functions of the center-of-mass energy

$$\delta(\sqrt{s}) = \left[ \frac{\int_{c_{min}}^{c_{max}} dc (d\sigma/dc)}{\int_{c_{min}}^{c_{max}} dc (d\sigma_0^H/dc)} - 1 \right] 100\% . \quad (12)$$

We took  $M_H = 250$  GeV, the value of photon energy threshold  $\omega_{th}=5$  GeV, and the angular range for photon detection  $-0.999 < c < 0.999$ . The dashed line represents the first order leading logarithmic correction. The solid line shows the values of the complete RC according to Eq. (8).

We see that the tree-level cross section dominates in the vicinity of  $\sqrt{s} \approx m_H$ . This region is appropriate for the  $\mu\bar{\mu}H$  vertex study. At the same time, the loop-induced cross section dominates in the region of  $\sqrt{s}$  which is far from  $m_H$  value. This region is appropriate, for example, for the  $t\bar{t}H$  vertex study.

## CONCLUSIONS

Muon colliders will deliver an excellent opportunity for the particle physics investigations.